MANIPULATIVES - CONSTRAINTS ON CONSTRUCTION?

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Abstract

The Secretary of Education (or other appropriate authority) has not determined that using manipulatives is either a sufficient or a necessary condition for meaningful learning. (Baroody, 1989, p. 4)

Statements extolling the virtues of manipulatives (concrete materials) for the learning of mathematics abound in curriculum documents, research literature and, even, textbook series. Concrete materials are often seen by teachers as the basis for mathematical learning. But is this a reasonable view?

In this paper, through a review of the literature concerning the use of concrete materials, we build up an historical view of the place of these materials in the learning and teaching of mathematics. Examples of situations in which the use of concrete materials has constrained children's mathematics learning are discussed along with the notion that materials introduce 'reality' to children's mathematical learning. Current research on children's thinking in the social and cultural contexts of their mathematics learning is used to help explain ways in which concrete materials can help and hinder this learning and how the very notion of manipulative might be expanded beyond concrete materials.

Introduction

In the past thirty years, the use of concrete materials in the learning and teaching of mathematics has been strongly advocated, particularly for children in the early childhood and primary school years (Australian Council for Educational Research, 1965; NSW Department of Education, 1969, 1972, 1989; Education Department of Western Australia, 1978; Booker, Irons & Jones, 1980; National Council of Teachers of Mathematics, 1980, 1989; Barry, Booker, Perry & Siemon, 1983 - 1994; Australian Education Council, 1991; Owens, 1994). Over this time, mathematics education research has exploded, with much of it focusing on children's learning of mathematics (Cockcroft, 1982; Briggs, 1984; Blane & Leder, 1988; Atweh & Watson, 1992; Grouws, 1992). That concrete materials play an important part in the development of children's mathematics has been accepted as a truism by the majority of teachers and mathematics educators.

In this same period, accepted approaches to the learning and teaching of mathematics have moved from an emphasis on transmission / absorption modes through activity-oriented approaches involving discovery and problem solving, to the present emphasis on children's construction of their own mathematical ideas and concepts. It would seem timely to examine more critically the place of concrete materials and manipulatives in the development of children's mathematical concepts in the light of these changing approaches to mathematics learning and teaching.

In Australia, the 1960s saw the strong acceptance, by educational systems, of 'structured' materials for the development of number concepts. The definition of 'structured materials' as "pieces of relatively simple equipment that have been carefully designed, so that the thinking of the child is directed towards mathematical relationships." (Australian Council for Educational Research, 1965, p. 5) reflects not only the perceived importance of the materials but also the particular learning style imposed by those materials. The most widely used structured material during this time was Cuisenaire. The advent of the 'new mathematics' and the development of a feeling among teachers that Cuisenaire was not appropriate for some number work, especially decimals, resulted in the further development of approaches using discrete materials (counters) during the seventies. During the late seventies, into the eighties and to the present day, there has been a gradual acceptance of Dienes' base 10 materials as the predominant number learning materials . Strong advocacy by mathematics educators (Booker, Irons & Jones, 1980), textbook series (*HBJ Mathematics* and *Rigby Mathematics*), and curriculum documents (NSW Department of Education, 1989; Australian Education Council, 1994) has ensured that base 10 materials have retained their popularity among teachers.

Currently, the use of concrete materials is not uniform across all year levels. There is a deal of evidence to suggest that such materials are likely to be used substantially in years K - 4 but that there continues to be a marked decline in their use in later years (Suydam, 1984, 1986; Gilbert & Bush, 1988).

Concrete Materials or Manipulatives?

In most instances, both of the terms 'concrete materials' and 'manipulatives' are taken to mean those "concrete models that incorporate mathematical concepts, appeal to several senses and can be touched and moved around by students." (Hynes, 1986, p. 11). These models could be either structured, such as Cuisenaire rods or Polydrons or environmental, such as paddle pop sticks or clay. Other authors use the term 'manipulatives' to incorporate both concrete and pictorial representations, including even images on computer screens (Touger, 1986; Sowell, 1989).

The notion of manipulatives in mathematics education has focused on the use of concrete materials to 'produce' the ideas assuming that the learners themselves do not bring with them ideas that can be manipulated. In this paper, we wish to expand the notion of manipulatives to encompass materials, diagrams, representations and **ideas** which the learner is able to use. In many cases such manipulatives will include concrete materials but this is by no means essential. What is essential is that the learner has a strong enough understanding of the manipulatives that he or she can use them to help develop further ideas. Similarly, Baroody (1989) suggests that:

> *... it does not follow ... that children must actively manipulate something* ***concrete*** *and reflect on* ***physical*** *actions to construct meaning. It does suggest that they should actively manipulate* ***something familiar*** *and reflect on these physical* ***or mental*** *actions. The particular medium ... may be less important than the fact that the experience is meaningful to pupils and that they are actively engaged in thinking about it.* (p. 5, Baroody's emphases).

Manipulatives have to be viewed as more than just concrete materials. A more viable definition, in the context of current theories and practices, would include any material, representation or idea which can be used by the learner to help construct new ideas. In discussing manipulatives and learning theories, and looking at the possible constraints to learning, the focus, in the past, has been on the materials themselves with little acknowledgement given to the learner's ideas. It is this focus which needs to be critically reconsidered.

Manipulatives and Theories of Learning

There can be no doubt that, in many instances, the use of manipulatives has proven helpful in assisting children to further develop their mathematical ideas (Thompson, 1992; Sowell, 1989; Bohan & Shawaker, 1994). Support for their use has come from curriculum developers, textbook writers and teachers as well as from learning theorists. In mathematics classes of the 1960s and 1970s, the key argument for the use of 'concrete materials' was the seminal work of Piaget related to stages of development. Piaget believed that children develop cognitively through stages and while concrete materials can be seen to be important at all of these stages, it was the first three where they were seen to be absolutely critical. The fact that the third stage was often considered to be completed around the age of ten or eleven years is a possible explanation why manipulatives are more accepted by teachers of K - 4 classes than they are by teachers of classes above Year 4.

The use of manipulatives also relies on a theoretical belief that the mathematics to be learned is somehow captured in the manipulatives and that what the learner has to do is to 'discover' this mathematics and transfer its material representation into a conceptual and symbolic representation. In fact, many studies advocating the use of manipulatives emphasise the importance of the learner being able to "bridge the very large gap between manipulatives and paper-and-pencil tasks" (Gluck, 1991, p. 10). In order to achieve this, "... it is critical that a close match be made between the way in which manipulatives are used and the expected outcomes at the symbolic stage." (Bohan & Shawaker, 1994, p. 246).

A direct result of this approach is the need for "teachers ... to orchestrate mathematical concept development very carefully to provide a smooth transition from the concrete to the abstract levels." (Heddens, 1986, p. 17) The following longer quote further reinforces this belief:

Resnick (1982) found that she was successful in teaching children to eliminate 'bugs' from their written subtraction algorithm only by ensuring that the written algorithm was an exact step-by-step record of the procedure which the child had carried out using base ten blocks. When this was done, the children did not develop any new bugs, and were able successfully to explain their subtraction procedure months later. Often, however, the practical work and the formalisation are only loosely connected ...

A child who does not see the connection between practical work and the formalised methods used on paper is little better off than the child who is taught the formalised method only by demonstration. This lack of strong connection between the practical work and the formalisation no doubt leads to the belief held by many teachers that 'children need lots of practical work, followed by practice'. It would seem that the intermediate step of making the link between concrete embodiment and formalisation is often missing in a child's experience. (Shuard, 1986, p. 84)

The above relationships between manipulatives and mathematics learning and teaching are clearly based upon a transmission / absorption approach (sometimes disguised as 'activity mathematics') whereby students seek to 'apprehend' the mathematical meaning which "is inherent in the words and actions of the teacher or in objects in the environment." (Cobb, 1988, p. 87). But how pertinent to current theories of learning is such an approach?

Curriculum documents (Australian Education Council, 1991; New South Wales Department of Education, 1989; National Council of Teachers of Mathematics, 1989) support a view that "learners construct their own meanings from, and for, the ideas, objects and events which they experience" (Australian Education Council, 1991, p. 16). This view raises important questions concerning the use of manipulatives in mathematics learning, at least as they have been traditionally employed. For example, how can we be sure that the 'correct' mathematical structure has been apprehended by a learner through the manipulation of certain materials? Or, how can we expect a learner to make sense of material which is more mathematically advanced than their current internal representation? How useful can an external representation be when the learner has to be told explicitly what is being represented? How limiting may manipulatives be when the learner's thinking is beyond the materials being used?

Constraints on Construction?

If we accept a constructivist theory of knowing and consider the consequences of this for learning and teaching, there are a number of potential constraints arising from the use of manipulatives as representations of the 'correct' mathematics to be developed.

Firstly, there is a question of **reality**. Often materials are used so that the mathematics to be developed can be given an embodiment which is real to the learner. But how viable is such an approach?

> *This notion of real world and mathematical abstract world, which for many teachers of young children is an algorithm for placing the child into a mathematically abstract context supported by some form of concrete representation, is a fundamental misconception. The reality of the world of concrete representation can be reality for a child only if it carries meaning for that child. Any abstractions that are then invoked by the child are a result of that child's construction of personal meaning out of the context. It is not a matter of holding a child using concrete apparatus until abstraction is possible, as if this were a linear procedure, but rather of always recognizing the need to slip in and out of representations in exactly the same way as particular examples are used to enhance the meaning of a generalization.* (Burton, 1990, p. 341)

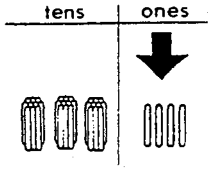
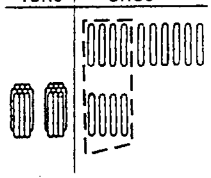
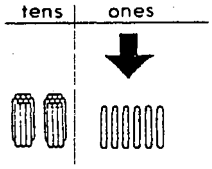
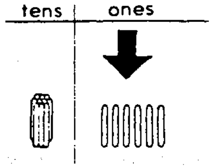
It is often assumed that the learner is able, through practice, either to apprehend the mathematical reality contained in the manipulatives or to accept what they are told the reality is, following interpretation by the teacher and / or their peers and, then, to transmit it into an internal representation of that reality. From a constructivist point of view, such external reality may not exist and the internal realities vary from learner to learner. Steffe (1992) has given the following succinct summary of this situation: "We don't find mathematics **in** realistic situations without **putting it there**." (p. 13).

A learner's **willingness** to become actively involved with manipulatives is a continuing concern. The basis for all learning is active involvement, particularly active mental involvement. Manipulatives can help stimulate this action. But who should decide when and what materials are to be used in the development of mathematical ideas or when these ideas are to be developed? Is it sufficient to suggest that children will use materials when they feel the need for them and cease using materials when they no longer feel the need? Wright (1992) suggests not.

> *A common response of teachers to the question of when children should cease using structured materials is to say that children should be allowed to use them until they show that they are ready to work without them. This approach ignores the point that children typically will continue to use a less advanced strategy unless there is some pressure (cognitive) to change.* (p. 129)

These suggestions need to be considered against the well known scenario of children using their fingers or rulers to help them calculate. This is a sign that the children need materials to undertake these tasks and they tend to use the most readily available. Short of surgically removing these materials from the child, what can teachers do?

Another possible area of constraint on a learner's construction of mathematics which may result from using concrete materials arises from the very **funnelling** of thoughts long held to be such an advantage. For example, what room is there for individual construction of approaches in the following links between materials and language presented by Barry, Booker, Perry & Siemon (1990, p. 59)?

Language	Materials	Recording
34 take away 18	tens | ones 	$\begin{array}{r} 3\ 4 \\ -1\ 8 \\ \hline \end{array}$
What do you take away first? (the ones). 4 ones; can you take away 8 ones? No. Trade 1 ten for 10 ones.	tens | ones 	$\begin{array}{r} 2\ 14 \\ \cancel{3}\ \cancel{4} \\ -1\ 8 \\ \hline \end{array}$
14 take away 8? (6).	tens | ones 	$\begin{array}{r} 2\ 14 \\ \cancel{3}\ \cancel{4} \\ -1\ 8 \\ \hline 6 \end{array}$
2 tens take away 1 ten? (1 ten).	tens | ones 	$\begin{array}{r} 2\ 14 \\ \cancel{3}\ \cancel{4} \\ -1\ 8 \\ \hline 1\ 6 \end{array}$
34 take away 18 is 16.		

The directed use of manipulatives with the preferred language and symbolic representation to be learned, while perhaps serving to link actions using the materials with 'correct' symbolic algorithms may do little to develop the learner's own mathematical constructions. Nonetheless, teachers may need to know such representations to facilitate children's mathematical learning.

This is a clear example of how changes in the acceptance of learning theories and practices require rethinking of the use of manipulatives.

Other issues which might be seen as constraints but which have not been considered here include the traditional view of **transfer**, whether manipulatives can be seen, by both teachers and learners, to be **culture-free** and their use **belief-** and **value-free** and the use of manipulatives by teachers.

Conclusion

The prime focus of this paper was to extend the notion of manipulatives beyond concrete materials to include diagrams, representations and ideas and to investigate the usefulness of this notion in the light of constructivist theories of learning. In discussing this, we have considered possible constraints for the development of mathematical concepts which may arise from the use of manipulatives. This paper represents a beginning which will hopefully stimulate further research into the effective use of this extended notion of manipulatives in classrooms reflecting constructivist theories of learning. Possible topics in such a research program are:

* community perception of the use of manipulatives;
* the role and acceptance of manipulatives in Years 4 - 12;
* reappraisal of the use of manipulatives in Years K - 4;
* availability and choice of manipulatives given to learners;
* the place of computers, calculators and other technologies as manipulatives;
* good questions and the use of manipulatives;
* use of manipulatives in all areas of mathematics;
* the relationship between manipulative use and visualisation in mathematics learning;
* critical appraisal of the use of manipulatives in current mathematics curriculum documents.

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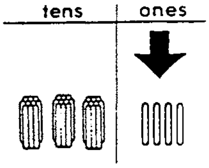
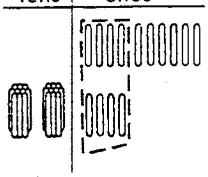
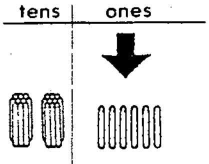
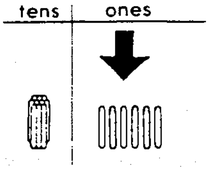
34 take away 18	tens | ones 	3 4 -1 8
What do you take away first? (the ones). 4 ones: can you take away 8 ones? No. Trade 1 ten for 10 ones.	tens | ones 	2 14 $\cancel{3}$ $\cancel{4}$ -1 8
14 take away 8? (6).	tens | ones 	2 14 $\cancel{3}$ $\cancel{4}$ -1 8 6
2 tens take away 1 ten? (1 ten).	tens | ones 	2 14 $\cancel{3}$ $\cancel{4}$ -1 8 1 6
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